

DISPERSION OF NONLINEAR VISCOPLASTIC MEDIA
IN TUBES

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Dispersion is considered of nonlinear viscoplastic media in circular tubes in the case of laminar motion. The problem is solved by using the approximate Taylor method. Results are shown of computer calculations for various values of the rheological parameters τ_0 , η , m , and n .

Mixing takes place in successive motion of mutually soluble fluids and a mixture convection zone is formed with physical characteristics which are dependent on concentrations. In the general case the dynamics of mixing systems depends on a multitude of factors; first of all, on the physical characteristics of the displaced and displacing medium (viscosities, densities, temperature, etc.) and on the completeness of mixing.

In 1953 an approximation was analyzed by Taylor of the simple case of displacement of viscous mixing fluids in capillary tubes in the case of laminar motion in which the mixing fluids possess the same properties and are completely soluble in each other. He established that the dispersion of viscous fluids in slow laminar flow is governed by the standard law of Fick's molecular diffusion; however, the mixing intensity is characterized by the effective dispersion coefficient

$$D = \frac{v_{av}^2 R^2}{48D_0^2} = \frac{Pe^2}{48},$$

which depends on the Péclet diffusion parameter. Subsequently, a correction to the solution was introduced by Aris [2] in the case of radial diffusion and diffusion parallel to the tube axis being of the same order. Quite recently Maron [3] analyzed the problem in a more general formulation by eliminating several assumptions of the Taylor method. In [4-6] the approach was generalized to non-Newtonian systems. In the present article an attempt is made to extend the Taylor's diffusion theory to nonlinear viscoplastic media described by the generalized rheological law [7]

$$\tau^n = \tau_0^n + (\eta\dot{\gamma})^m. \quad (1)$$

The assumption that the two media following each other possess the same rheological parameter enables one to assume that the velocity distribution is independent of the distribution of the media and is given by

$$v_1'(r) = \frac{1}{\eta} \sum_{k=0}^m (-1)^k C_m^k \frac{n}{m+n-k} \left(\frac{\Delta P}{2l} \right)^{\frac{m-k}{n}} \tau_0^{\frac{k}{n}} \left(R^{\frac{m+n-k}{n}} - r^{\frac{m+n-k}{n}} \right), \quad r_0 \leq r \leq R; \quad (2)$$

$$v_0'(r) = \frac{1}{\eta} \sum_{k=0}^m (-1)^k C_m^k \frac{n}{m+n-k} \left(\frac{\Delta P}{2l} \right)^{\frac{m-k}{n}} \tau_0^{\frac{k}{n}} \left(R^{\frac{m+n-k}{n}} - r_0^{\frac{m+n-k}{n}} \right), \quad 0 \leq r \leq r_0. \quad (3)$$

If the diffusion parallel to the axis is ignored, then the concentration of the displacing fluid is determined by a system of equations which in their dimensionless form are given by

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$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial C_i}{\partial \rho} \right) = \frac{\partial C_i}{\partial \tau'} + v_i \frac{\partial C_i}{\partial \xi^2}, \quad (4)$$

where $i = 0$ for the region $0 \leq \rho \leq \rho_0$, $i = 1$ for the region $\rho_0 \leq \rho \leq 1$ with the boundary conditions

$$\left. \frac{\partial C_i}{\partial \rho} \right|_{\rho=1} = 0, \quad C_0|_{\rho=\rho_0} = C_1|_{\rho=\rho_0}, \quad C_0|_{\rho=0} \neq \infty.$$

By following the procedure of [5] one can obtain an expression for the effective dispersion coefficient:

$$\begin{aligned} D = & 2 \left\{ \frac{S - Pe}{4} \left[\frac{(S - Pe)(1 - \rho_0^2)^2}{4} - \sum_{k=0}^m B_m^k \frac{n}{m + 5n - k} \times \right. \right. \\ & \times \left(1 - \rho_0 \frac{m+5n-k}{n} \right) + \rho_0^2 \sum_{k=0}^m B_m^k \frac{n}{m + 3n - k} \left(1 - \rho_0 \frac{m+3n-k}{n} \right) \left. \right] + \\ & + (Pe - S) \sum_{k=0}^m B_m^k \left(\frac{n}{m + 3n - k} \right)^2 \frac{n}{m + 5n - k} \left(1 - \rho_0 \frac{m+5n-k}{n} \right) + \\ & + \sum_{k=0}^m \sum_{i=0}^m B_m^k B_m^i \left(\frac{n}{m + 3n - k} \right)^2 \frac{n}{2m + 6n - k - i} \times \\ & \times \left(1 - \rho_0 \frac{2m+6n-k-i}{n} \right) + \frac{S - Pe}{2} \sum_{k=0}^m B_m^k \left(\frac{n}{m + 3n - k} \right)^2 \times \\ & \times \rho_0 \frac{m+3n-k}{n} \left(1 - \rho_0^2 \right) - \sum_{k=0}^m B_m^k \left(\frac{n}{m + 3n - k} \right)^2 \rho_0 \frac{m+3n-k}{n} \times \\ & \times \sum_{k=0}^m B_m^k \frac{n}{m + 3n - k} \left(1 - \rho_0 \frac{m+3n-k}{n} \right) + \frac{(v_0 - Pe) \rho_0^2}{8} (S - Pe)(1 - \rho_0^2) - \frac{(v_0 - Pe) \rho_0^2}{4} \times \\ & \times \sum_{k=0}^m B_m^k \frac{n}{m + 3n - k} \left(1 - \rho_0 \frac{m+3n-k}{n} \right) + \left[\frac{Pe - S}{2} + \sum_{k=0}^m B_m^k \frac{n}{m + 3n - k} \right] \times \\ & \times \left[(S - Pe) \left(-\frac{1}{2} \ln \rho_0 + \frac{\rho_0^2}{4} - \frac{1}{4} \right) + \ln \rho_0 \sum_{k=0}^m B_m^k \frac{n}{m + 3n - k} + \right. \\ & \left. + \sum_{k=0}^m B_m^k \left(\frac{n}{m + 3n - k} \right)^2 \left(1 - \rho_0 \frac{m+3n-k}{n} \right) \right] \left. \right\} + \frac{(v_0 - Pe)^2}{8} \rho_0^4, \quad (5) \end{aligned}$$

where

$$\begin{aligned} S &= \sum_{k=0}^m B_m^k; \\ B_m^k &= (-1)^k \frac{R}{\eta D_0} \frac{m+2n-k}{n} C_m^k \frac{n}{m+n-k} \left(\frac{\Delta P}{2l} \right)^{\frac{m-k}{n}} \tau_0^{\frac{k}{n}}. \end{aligned}$$

In particular, by modifying the rheological parameters m and n expressions are obtained for the effective dispersion coefficient in the case of non-Newtonian fluids which follow the laws of Hershel, Caisson, Ostwald de Vallé, Bingham, and also of viscous media.

Similarly as in [1], one can obtain for the length L of the displacement zone (that is, the portion of the tube between the sections with average concentrations 90% and 10%),

$$L = 3.62 \sqrt{D\tau'}. \quad (6)$$

Numerical computer calculations were carried out for various values of the limiting dynamic displacement stress τ_0 and the structural viscosity η if the parameters of the systems in motion vary within the limits m, n

TABLE 1. Results of Computer Calculations of the Dimensionless Length of the Mixture Zone Using Formulas (5) and (6) for $R = 0.124 \text{ m}$, $D_0 = 1 \cdot 10^{-9} \text{ m}^2/\text{sec}$

$m=n$	$\tau_0 = 10 \text{ N/m}^2, \eta = 0.04 \text{ N}\cdot\text{sec/m}^2$			$\tau_0 = 6 \text{ N/m}^2, \eta = 0.015 \text{ N}\cdot\text{sec/m}^2$		
	ρ_0	$(L/\sqrt{\tau}) \cdot 10^{-7}$	$Pe \cdot 10^{-7}$	ρ_0	$(L/\sqrt{\tau}) \cdot 10^{-7}$	$Pe \cdot 10^{-7}$
1	0,71	4,181	18,683	0,77	3,283	18,073
	0,85	0,561	4,597	0,85	0,897	7,356
	0,90	0,165	1,997	0,90	0,264	3,194
2	0,38	36,078	24,446	0,45	45,861	22,122
	0,50	24,923	9,119	0,50	39,786	14,591
	0,80	12,415	0,331	0,80	19,863	0,530
3	0,17	186,090	36,812	0,21	231,132	34,131
	0,30	95,117	7,202	0,30	152,183	11,523
	0,60	43,006	0,228	0,60	68,802	0,365
4	0,07	1011,528	58,958	0,10	1045,291	41,466
	0,20	290,812	3,670	0,20	465,382	5,872
	0,50	107,834	0,055	0,50	168,734	0,077

$m = 1, 2, 3, 4, 5, 6$; examples are shown in Table 1 of the results of calculations on the BÉSM-6 computer for the dimensionless length $L/\sqrt{\tau}$ of the mixture zone depending on the Péclet diffusion number for various rheological parameters.

An analysis of the data shows that with the parameters m and n increasing, that is, with a reduction in the rheological properties of fluids [7], the dimensionless length of the mixture zone grows. The ratio τ_0/η also has an effect on the dispersion progress so that for $m = n = 1$ (viscoplastic media) the length of the mixture zone becomes shorter if the above ratio increases; for $m = n > 1$ the converse is true.

NOTATION

τ , dynamic displacement stress; τ_0 , limiting dynamic displacement stress; η , structural viscosity; $\dot{\gamma}$, displacement rate gradient; m, n , rheological parameters determined experimentally; v_i^1 , velocity distribution in a tube; v_{av} , average flow rate; C_m^k , the number of combinations; $\Delta P/2l$, pressure drop in tube length; R , tube radius; r_0 , radius of elastic core; r , coordinate in radial direction; x , coordinate parallel to tube axis; t , time; $\rho = r/R$, dimensionless coordinate in radial direction; $\rho_0 = r_0/R$, dimensionless radius of elastic core; $\xi = x/R$, dimensionless coordinate in the direction of tube axis; $\tau^1 = D_0 t/R^2$, dimensionless time variable; $v_i^1 = v_i^1 R/D_0$, dimensionless velocity distribution; D_0 , molecular diffusion coefficient; D , effective dispersion coefficient; $Pe_{av} = v_{av} R/D_0$, Péclet diffusion parameter; C_i , average concentration across tube section; L , length of mixture zone.

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